

DECISION OF COURT CASE BASED ON HYPOTHESIS TESTING

M. S. Rahman¹
M.R.K. Khadem²

ABSTRACT

When a person comes to a court charged with a crime, a jury must decide whether the person is innocent or guilty. Even though the person is charged with the crime, at the beginning of the trial and until the jury declares otherwise based on evidence, the accused person is assumed to be innocent. This paper deals with introducing a general method of deciding for any court case based on a non-parametric statistical hypothesis testing procedure. An appropriate null and alternative hypotheses have been stated, a test statistic has been suggested which follows a binomial probability distribution. The decision rule and test method have similarity with the sign test. Applications of this method are provided.

KEYWORDS: Court case, Hypothesis, Binomial distribution, Sign test, Nonparametric.

INTRODUCTION:

Let us consider a situation, after a criminal activity, the victim's family filed a case in the court suspecting someone. Then the accused person comes to the court for justice. The court or a jury must decide whether the person is innocent or guilty based on the evidence presented in the court relating to the case. Even though the person is charged with the crime, at the beginning of the trial and until the jury declares otherwise, the accused is assumed to be innocent. Only if overwhelming evidence of the person's guilt convinced the jury to declare him guilty, otherwise the person is considered innocent. This article introduced a hypothesis testing method of deciding for any court case lodged anywhere in the world. In hypothesis testing first, we need to setup null (H_0) and alternative hypotheses (H_1).

Boulanouar (2017) stated that a hypothesis is a tentative statement that makes a prediction, based on knowledge and research, about the relationship between two or more variables and proposing a possible solution to a problem or an explanation of some phenomenon. This hypothesis concept also discussed by many authors such as Ary et.al. (1984), Creswell (2014), and Maxwell (2013). Therefore, in a court case, the following hypotheses can be considered.

H_0 : The accused person or dependent is innocent.

H_1 : The accused person or dependent is guilty.

In the jury trial there are two types of possible errors: (i) the person is innocent, but the jury declares him guilty based on false evidence, and (ii) the person is guilty, but the jury declares him innocent based on insufficient evidence. In the judicial system, the first error is considered more serious than the second error. Because giving punishment to an innocent person is a more serious issue than declaring a guilty person as innocent. Keeping this in mind, we fix the probability of type I error usually denoted by α as low as possible, 1% or, less. These two errors along with the correct decisions are shown in the following table.

Table 1: Type I and Type II Error

	The person is innocent	The person is guilty
Jury declares the person is innocent	Correct Decision	Type II Error
Jury declares the person is guilty	Type I Error	Correct Decision

Here type I error indicates, the accused person or dependent is innocent but based on false evidence the jury convicted him. That is, α is the probability of giving punishment to an innocent dependent, which may be called as a dependent's risk. Type II error indicates, the dependent is guilty but based on the lack of evidence the jury declared him innocent and the probability of this error is usually denoted by β . That is, β is the probability of declaring a guilty person as innocent, which may be called as accuser's risk. In MP (most powerful) test we are maximizing $(1-\beta)$ that is minimizing β . That is minimizing the accuser's risk which is desirable. α has several names: Probability of type I Error, Level of Significance, Probability of rejection region when H_0 is true, Size of a test, Producer's risk, or, Dependent's risk. Similarly, β also has several names: Probability of type II Error, Consumer's risk, Accuser's risk, or, Probability of acceptance region when H_1 is true.

Burtis and Kobayashi (2017) studied the relationship between legal decision rules and thresholds of statistical significance level α . Their analysis demonstrates how a choice of either a statistical significance threshold or a legal standard represents alternative and often inconsistent ways to trade off error costs, and that threshold based on fixed significance levels are not generally consistent with optimal legal rules. They also showed how the two thresholds can be reconciled by replacing fixed significance levels with likelihood ratio tests.

In this article, an attempt has been made to develop a new non-parametric method of deciding on any court case. Section 2 deals with materials and methods. Two practical examples of the proposed method are given in Section 3. Section 4 is the conclusion.

2. MATERIALS & METHODS:

2.1 DATA: The data consist of recording the opinions of n individuals/witnesses participating in the study. We may use a plus (+) sign if the witness expresses the accused person is innocent and a minus (-) sign if the witness expresses the accused person is guilty. Since the data are recorded in terms of plus and minus signs whatever be the parent population distribution, this is a nonparametric test and usually called the sign test. We ignore the neutral opinion that is if the opinion of the witness indicates that the accused person is neither innocent nor guilty. In hypothesis testing we consider five steps: (i) setup H_0 and H_1 , (ii) select a test statistic, (iii) choose the level of significance (α), (iv) setup decision rule, and (v) conclusion.

2.2 TEST STATISTIC:

The test statistic for the sign test is

$$T = \text{total number of + signs}$$

Here the opinions of n individuals/witnesses participating in the study are independent and each opinion takes one of two possible values plus (+) sign or, minus (-) sign. Therefore, T follows a binomial distribution which doesn't depend on parent population distribution indicating a non-parametric test.

2.3 DECISION RULE: There is a possibility that some witness may not clearly express the accused person is innocent or guilty which can be considered as a neutral witness.

First, we must avoid/omit all neutral opinions and we consider

$$n = \text{total number of plus and minus signs.}$$

Reject H_0 if $T \leq t$ or $T \geq n - t$, at a level of significance α , where t is the value of the random variable T corresponding to $\alpha/2$ for two-sided test and α for the one-sided test in binomial probability distribution table [Neave and Wothington (1988)]. Otherwise, we accept H_0 . That is the accused person is innocent or guilty can be decided based on this hypothesis testing procedure. This can be used for deciding on any court case in any country of the world where fair and independent justice system exist.

If n is greater than 20, the binomial distribution approximated to normal and in that case, our t is $t = \frac{n + Z_{\alpha/2}\sqrt{n}}{2}$, where $Z_{\alpha/2}$ is obtained from the normal distribution table [Conover (1980)].

3. APPLICATION: The proposed method has been applied in the following two examples.

EXAMPLE 1: We have considered an incident mentioned in the Holy Qur'an. The verses 25-28 from the chapter 12, Surah Yusuf (Joseph) of Holy Qur'an describe an incident between the Lady of the house and Yusuf: "And they both raced to the door, and she tore his shirt from the back, and they found her husband at the door. She said, "What is the recompense/penalty of one who intended/wished evil for your wife but that he be imprisoned or a painful punishment (Verse 25) [Ali (2001)]? [Joseph] said, "It was she who sought to seduce me." And a witness from her family testified. "If his shirt is torn from the front, then she has told the truth, and he is of the liars" (Verse 26) [Ali (2001)]. "But if his shirt is torn from the back, then she has lied, and he is of the truthful" (Verse 27) [Ali (2001)]. "So when her husband saw his shirt torn from the back, he said, "Indeed, it is of the woman's plan. Indeed, your plan is great" (Verse 28) [Ali (2001)].

Boulanouar (2017) provides a comparative overview of the conventional literature on hypothesis testing, and this incident from the Quran, to show how the closeness of the two methods. The Qur'an as a text is over 1400 years old. Boulanouar (2017) stated that in terms of methodological presentation in general, a favorite style of the Quran is narrative. After whatever took place inside the castle/house (verses 23-24) [Ali (2001)], the first two characters of the case, the lady and Yusuf, raced with one another to the door - but for different reasons. She, while trying to (catch and) force him into acting as she wished, pulled his shirt from behind, thus tearing it. Up until then, they were alone, so nobody could have seen or witnessed what happened between them. Then, on the other side of the door, they met her husband, the third character in the case, who is also the Master of the house. At this moment, the wife of the master, the accuser, claimed that Yusuf tried to commit an odious act against her. Then she added, "so what is the recompense for such a perpetrator be imprisoned or a painful punishment?" Yusuf, however, defending himself against this accusation, stated his version of the event: it was her who tried seducing him. Here, two propositions have been stated, each claiming to be true by each from a different point of view: that of the accuser and that of the accused. In summary, party 1 accuses party 2 of wishing to do harm/evil to her, and party 2 denies it. The Master of the house is presented with two opposing possibilities or alternatives, while only one of them can be true, however, in his capacity of judge, in this case, he is unable to work out which of the two propositions is true and thus he cannot adjudicate on the case. These two propositions are like null and alternative hypotheses. There comes a fourth and last character, identified as an expert witness. The Qur'an has explicitly documented the expert witness as being "of her own folk" i.e. from the family of the Lady of the house, the party that will eventually be identified as guilty (i.e. the witness will provide testimony against her kinswoman). Had the witness been from the family of the party the sentence was made in favor of, it would be easy to reject the testimony. The expert witness, accepted by all 3 characters, after examining the case, devises a key and systematic procedure to help the Master bring out the truth, and successfully adjudicate. Boulanouar (2017) compared Hypothesis testing (HT) method with this Quranic incident and concludes that Hypothesis testing method is a much older framework than that may be commonly believed. In this article, we have applied our proposed test to this Quranic incident. This procedure consisted of setting up two explicit hypotheses.

H_0 : The accused person (Yusuf) is innocent.

H_1 : The accused person (Yusuf) is guilty.

There are six observations

X_1 = Yusuf and the lady raced one another to the door.

X_2 = The allegation of the lady.

X_3 = The defending voice of Yusuf.

X_4 = The method proposed by an expert witness (Shirt was torn from the back or from the front).

X_5 = The expert witness is from the family of the lady or from the family of Yusuf.

X_6 = Neutrality of the argument, that is whether the method proposed by the expert witness is accepted by all parties involved in this case.

We use a plus (+) sign if the observation indicates the accused person is innocent and a minus (-) sign if the observation indicates the accused person is guilty.

+, -, +, +, +, 0

Here X_1 is positive because Yusuf wanted to get out of the house to avoid the odious act and that so why was running towards the door. X_2 is negative because it indicates Yusuf is guilty. Similarly, we have assigned positive or, negative signs for X_3 , X_4 and so on.

Therefore, test statistic $T = 4$ and $n=5$.

If we consider $\alpha = 0.01$, the critical value is 0 from statistical Table [Neave and Worthington (1988)].

Hence, the decision rule is

Reject H_0 if $T \leq 0$ or, $T \geq 5$,

The decision does not reject H_0 . That is Yusuf is innocent.

EXAMPLE 2: The proposed test method can also be applied in a situation of selecting the most probable option out of two possible options. Here we have considered another incident 'Isra and Miraj' mentioned in the Holy Qur'an [Ali (2001)]. '**Isra**' means '**journey at night**' and '**Miraj**' means to "**ascend**". Therefore, the Isra and Miraj event represent the whole journey of the Prophet (PBUH) from His house to the higher skies and return [Al-Mubarkpuri (2014)]. The Verse 1 from the Chapter 17, Surah Bani Israel, stated "Glory to (Allah) Who did take His Servant for Journey by night from the Sacred Mosque to the Farthest Mosque whose precincts We did bless – in order that We might show him some of Our Signs: for He is the one Who heareth and seeth (all things)". Further to that, the Verses 7-18 from the Chapter 53, Surah An-Najm, stated "While he was in the highest part of the horizon: Then he approached and came closer, And was at a distance of but two bow-lengths or (even) nearer; So did (Allah) convey the inspiration to His Servant (conveyed) what He (meant) to convey. The (Prophet's) (mind and) heart in no way falsified that which he saw. Will ye then dispute with him concerning what he saw? For indeed he saw him at a second descent near the Lote-tree beyond which none may pass: near it is the Garden of Abode. Behold, the Lote-tree was shrouded (in mystery unspeakable!) (His) sight never swerved nor did it go wrong! For truly did he see of the Signs of His Lord the Greatest?" [Ali (2001)].

These ayahs refer to the peak of Miraj where Prophet Muhammad (PBUH) met Allah Almighty (physically or, spiritually). Every Muslim is aware of this incident happened during the night, however, there are many important things happened on that night of Isra and Miraj those are mentioned in different authentic Hadith. Some of them are as follows:

The story of the Miraj goes that the beloved Prophet (PBUH) was asleep one night in Makkah when he was awakened by the angel Gabriel(as) who cleansed his heart before riding him on an angelic horse named Buraq [Al-Mubarkpuri (2014)]. In Sahih Muslim (Vol. 1, p. 101) it was stated that the Prophet (PBUH) was brought a Buraq who is an animal white and long, larger than a donkey but smaller than a mule, who would place his hoof at a distance equal to the range of vision [Al-khattab (2007)]. The Prophet (PBUH) mounted it and came to the Temple (Bait-ul Maqaddis in Jerusalem), then tethered it to the ring used by the previous prophets.

The journey from Makkah to Jerusalem is known as al-Isra. At Jerusalem, the Prophet (PBUH) was tested in the following way by Gabriel(as).

Sahih al-Bukhari (Vol. 6, p. 196) stated that Allah's Apostle was presented with two cups, one containing wine and the other milk on the night of his night journey at Jerusalem. He looked at it and took the milk. Gabriel said, "Thanks to Allah Who guided you to the Fitra (i.e. Islam); if you had taken the wine, your followers would have gone astray" [Khan (1996)]. After this began al-Mi'raj. The Prophet (PBUH) passed the sea of kawthar, the sea of "abundance", and then met various prophets, from Adam(as) to Abraham(as), as well as a variety of angels as he passed through the seven heavens. After this Gabriel(as) took him to the heavenly lote-tree on the boundary of the heavens before the throne of Allah. In Sahih al-Bukhari (Vol. 5, p. 147) it was stated that then the Prophet (PBUH) was made to ascend to Sidrat-ul-Muntaha (i.e. the lote-tree of the utmost boundary). Behold! Its fruits were like the jars of Hajr (i.e. a place near Medina) and its leaves were as big as the ears of elephants. Gabriel said, "This is the lote-tree of the utmost boundary" [Khan (1996)]. Gabriel(as) and Buraq could go no further but the Beloved Prophet(PBUH) went on to the presence of Allah where he was commanded to order the Muslims to pray fifty times a day: Sahih al-Bukhari (Vol. 1, p. 213) stated that then Allah enjoined fifty prayers on his followers. When he returned with this order of Allah, he passed by Moses who asked him, "What has Allah enjoined on your followers?" The Prophet (PBUH) replied, "Allah has enjoined fifty prayers on them". Moses said, "Go back to your Lord (and appeal for reduction), your followers will

not be able to bear it" [Khan (1996)]. The Prophet (PBUH) went back and forth between Allah and Moses(as) till the prayers were reduced to five per day. Moses(as) then told him to seek yet a further reduction but the prophet stopped at this point and answered Moses: Ishaq (1998) stated that the Prophet (PBUH) replied that he had been back to his Lord and asked him to reduce the number until he was ashamed, and he would not do it again. Allah then said whoever observed the five times of prayer daily would receive the reward of fifty prayers. The Prophet(PBUH) then saw some of the delights of paradise as he returned to Gabriel(as) and Buraq and then beheld the torments of the damned before going back to his home in Makkah that same night. This, briefly, is the narrative of the ascent. The Qur'an also supports this miraculous journey.

On the way back, he came upon some caravans that were also traveling to Mecca. The Prophet (PBUH) said: "I saw the caravan of the tribe of So-and-so as I was coming back. They had lost one of their camels and were searching for it everywhere. I reached their mounts and there was no one with them. I found a water bottle and I drank from it." A few weeks later, facts confirmed some elements of his account, for instance, the arrival of caravans whose coming he had announced (having seen them on his way back) and of which he had given a precise description.

The Prophet (PBUH) continued: "Then I reached the caravan of the tribe of So-and-so in such-and-such a place. I saw a red camel carrying one black container and one white one. When I came face to face with the caravan there was a stampede and that camel fell and its freight broke. Then I reached the caravan (not previously mentioned) of the tribe of So-and-so in al-Tan'im. It was headed by a grayish camel on which was a black hair-cloth and two blackish containers and here are the (three) caravans about to reach you from the mountain pass."

A few days later the caravan arrived in Makkah. Then the people meet the riders and asked them: "Did you lose a camel?" They said yes. They asked the second caravan: "Did one red camel of yours shatter her freight?" They said yes. They asked (the first caravan): "Did anyone lose a water bottle?" One man said: "I did, by Allah, I had prepared it but none of us drank it nor was it spilled on the ground!" At this, they said: "al-Walid spoke the truth." And Allah revealed the verse 60 from the chapter 17, Surah Al-Mumtahana: "We appointed not the vision which we showed you but as a test for mankind".

All Muslims believe the event has occurred, but there is controversy about the way it occurred physically or spiritually. Our proposed test method can be applied to this Quranic incident for deciding. The appropriate hypotheses to be considered here are as follows:

H_0 : The Isra and Miraj event occurred physically.

H_1 : The Isra and Miraj event occurred spiritually.

There are five observations

X_1 = Total distance traveled.

X_2 = Total time spent.

X_3 = Transport is associated with physical Journey.

X_4 = Observing something (farthest mosque) physically can be described and verified.

X_5 = Observing something on the way physically can be described and verified.

We use a plus (+) sign if the observation indicates the Isra & Miraj event happened physically and a minus (-) sign if the observation indicates the Isra & Miraj event happened spiritually.

-, -, +, +, +

Therefore, test statistic $T = 3$ and $n=5$.

If we consider $\alpha = 0.01$, the critical value is 0.

Hence, the decision rule is

Reject H_0 if $T \leq 0$ or, $T \geq 5$,

The decision does not reject H_0 . That is the Miraj event happened physically.

4. CONCLUSION:

We introduce a new method of deciding on any court case based on a non-parametric statistical hypothesis testing procedure named sign test. This method has been applied to two important incidents mentioned in the Holy Qur'an and obtained similar results which also mentioned in the Holy Qur'an. These incidents have been illustrated by several researchers such as Boulanouar (2017), Burtis and Kobayashi (2017) etc. using a different approach but they didn't use statistical test for deciding on such situation. This method can also be applied anywhere in the world by any country for deciding in any court case if the people of that country have the freedom of expressing their opinion freely and independently. That is the country should have a fair and independent justice system, but in a real sense, many countries do not have it. This is one limitation of applying this method.

REFERENCES:

- Ali, A.Y. (2001), *The Holy Qur'an* translated by Abdullah Yusuf Ali, Wordsworth Editions Ltd, 5th edition, Birmingham
- Al-khattab, N. (2007), *English Translation of Sahih Muslim (7 volumes set)*, Darussalam, A Global leader in Islamic books, New York
- Al-Mubarkpuri, S.R. (2014), *Al-Isra and Al-Miraj, The inspiring Journey*, 2nd edition, onislam.net
- Ary, D., Jacobs, L.C. and Razavieh, A. (1984), "Introduction to Research in Education". Holt, Rinehart and Winston, Inc.
- Boulanouar, Z. (2017), *The Islamic Origins of Hypothesis Testing*, IRTI (Islamic Research and Training Institute) Research Seminars, Umm Al Qura University, Saudi Arabia, 25 October 2017
- Burtis, M.M. and Kobayashi, B. H. (2017), Error costs, statistical significance, and legal decision rules, <https://www.law.umich.edu/centersandprograms/lawandeconomics/workshops/Documents/paper11>
- Conover, W.J. (1980), *Practical Nonparametric Statistics*, Second Edition, John Wiley & Sons, New York
- Creswell J. W. (2014), "Research Design: Qualitative, Quantitative, and Mixed Methods Approaches" 4th edition, SAGE Publications, London
- Ishaq, I. (1998), *The life of Muhammad*, 13th edition, Oxford University Press, New York.
- Khan, M. M. (1996), *The English Translation of Sahih Al Bukhari with the Arabic Text (9 volumes set)*, 1st Edition, Al-Saadawi publications
- Maxwell, J.A. (2013), *Qualitative Research Design-An Interactive Approach*. 3rd edition, Sage Publications.
- Neave, H.R. and Worthington, P.L. (1988), *Distribution Free tests*, Unwin Hyman, London, Sydney

M. S. Rahman¹
¹*Dept of OMBS, CEPS,
Sultan Qaboos University
Muscat, Sultanate of Oman
srahman@squ.edu.om*

M.R.K. Khadem²
²*Dept of MIE, CE,
Sultan Qaboos University
Muscat, Sultanate of Oman
khadem@squ.edu.om*